

String or M Theory Axion as a Quintessence

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Abstract

A slow-rolling scalar field ($Q \equiv$ Quintessence) with potential energy $V_Q \sim (3 \times 10^{-3} \text{ eV})^4$ has been proposed as the origin of accelerating universe at present. We investigate the effective potential of Q in the framework of supergravity model including the quantum corrections induced by generic (non-renormalizable) couplings of Q to the gauge and charged matter multiplets. It is argued that the Kähler potential, superpotential and gauge kinetic functions of the underlying supergravity model are required to be invariant under the variation of Q with an extremely fine accuracy in order to provide a working quintessence potential. Applying these results for string or M -theory, we point out that the heterotic M -theory or Type I string axion can be a plausible candidate for quintessence if (i) it does not couple to the instanton number of gauge interactions not weaker than those of the standard model and (ii) the modulus partner $\text{Re}(Z)$ of the periodic quintessence axion $\text{Im}(Z) \equiv \text{Im}(Z) + 1$ has a large VEV: $\text{Re}(Z) \sim \frac{1}{2\pi} \ln(m_{3/2}^2 M_{\text{Planck}}^2 / V_Q)$. It is stressed that such a large $\text{Re}(Z)$ gives the gauge unification scale at around the phenomenologically favored value $3 \times 10^{16} \text{ GeV}$. To provide an accelerating universe, the quintessence axion should be at near the top of its effective potential at present, which requires a severe fine tuning of the initial condition of Q and \dot{Q} in the early universe. We discuss a late time inflation scenario based on the modular and CP invariance of the moduli effective potential, yielding the required initial condition in a natural manner if the Kähler metric of the quintessence axion superfield receives a sizable nonperturbative contribution.

I. INTRODUCTION AND SUMMARY

Recent measurements of the luminosity-red shift relation for Type Ia supernovae suggest that the Universe is accelerating and thus a large fraction of the energy density has negative pressure [1]. A nonvanishing cosmological constant (vacuum energy density) would be the simplest form of dark energy density providing the necessary negative pressure. However there is a widespread prejudice that the correct, if exists any, solution to the cosmological constant problem [2] will lead to an exactly vanishing vacuum energy density. For instance, within the framework of the Euclidean wormhole mechanism [3] which has been proposed sometime ago as a solution to the cosmological constant problem, the vacuum energy density appears to exactly vanish if all fields (except for the spacetime metric) were settled down at their true vacuum expectation values (VEVs).

Recently quintessence in the form of slow-rolling scalar field has been proposed as an alternative form of dark energy density with negative pressure [4]. The true minimum of the quintessence potential is presumed to vanish, i.e. $(V_Q)_{\min} = 0$, however the present value of Q is displaced from the true minimum, providing

$$V_Q \sim (3 \times 10^{-3} \text{ eV})^4, \quad (1)$$

and a negative pressure with the equation of state:

$$\omega = \frac{p_Q}{\rho_Q} = \frac{\frac{1}{2}\dot{Q}^2 - V_Q}{\frac{1}{2}\dot{Q}^2 + V_Q} \lesssim -0.5, \quad (2)$$

while obeying its equation of motion:

$$\ddot{Q} + 3H\dot{Q} + \frac{\partial V_Q}{\partial Q} = 0. \quad (3)$$

In order to have negative pressure, we then need the following slow-roll condition:

$$\left| \frac{\partial V_Q}{\partial Q} \right| \lesssim \left| \frac{V_Q}{M_P} \right|, \quad (4)$$

where $M_P = M_{Planck}/\sqrt{8\pi} = 2.4 \times 10^{18} \text{ GeV}$ denotes the reduced Planck scale. This slow-roll condition indicates that the typical range of Q is of order M_P or at least *not* far below M_P .

Given the assumption of $(V_Q)_{\min} = 0$, one can in principle compute the quintessence potential V_Q once the particle physics model for Q is given. However the required size of V_Q is extremely small compared to any of the mass scales of particle physics, and thus an utmost question is how such a small V_Q can arise from realistic particle physics models. It must be stressed that V_Q is required to be small *over* a range of Q which is essentially of order M_P , *not only* for a particular value of Q . As a result, unless there were a symmetry which would enforce V_Q to be flat enough, V_Q suffers from the fine tuning problem much more severe than the conventional cosmological constant problem associated with the smallness of the potential energy at its minimum [5]. In fact, the most natural candidate for a light scalar field whose typical range of variation is of order M_P is the string or M -theory moduli

multiplets describing the (approximately) degenerate string or M -theory vacua. It is thus quite tempting to look at the possibility that Q corresponds to a certain combination of the string or M -theory moduli superfields. In this paper, we wish to explore this possibility and point out that the heterotic M -theory or Type I string axion can be a plausible candidate for quintessence if its modulus component has a large VEV and its Kähler metric receives a sizable nonperturbative contribution.

To make the discussion more rigorous, we provide in section II a careful analysis of the low energy effective potential of generic quintessence¹ in the framework of 4-dimensional effective supergravity model which is presumed to describe the dynamics of Q at high energy scales. We include both the perturbative and nonperturbative corections to V_Q induced by generic (nonrenormalizable) couplings of Q with the gauge and charged matter multiplets in the model. It is then argued that the Kähler potential (K), superpotential (W) and gauge kinetic functions (f_a) of the underlying supergravity model are required to be invariant under the variation of Q with an extremely fine accuracy, which implies as one of its consequences that non-derivative couplings of Q are extremely suppressed and so there is no Q -mediated long range force. Also estimated are the size of Q -invariance breaking for various terms in K , W and f_a which would produce the correct value of V_Q .

Applying these results for string or M -theory, we show in section III that the heterotic M -theory or Type I string axion can be a plausible candidate for quintessence. The quintessence axion may correspond to the heterotic M -theory axion arising from the three form field of 11-dimensional supergravity on a manifold with boundary [6] or to the Type I string axion arising from the R-R two form field [7]. To avoid a too large potential energy, it is required that the quintessence axion does *not* couple to the instanton number of gauge interactions *not* weaker than those of the standard model, particularly not to those of the QCD and even stronger hidden sector gauge interactions. Then the effective potential of heterotic M -theory axion is mainly induced by the membrane instantons wrapping the 2-cycle of the internal 6-manifold and stretched along the 11-th dimension, while that of Type I string axion is induced by $D1$ or $D5$ instantons wrapping the 2 or 6-cycle. The resulting axion potential is estimated as

$$V_Q \sim e^{-2\pi\langle\text{Re}(Z)\rangle} m_{3/2}^2 M_P^2 \cos[2\pi\text{Im}(Z)], \quad (5)$$

where $\text{Re}(Z)$ is the modulus partner of the periodic quintessence axion $\text{Im}(Z) \equiv \text{Im}(Z) + 1$ and $m_{3/2}$ is the gravitino mass. Thus the quintessence potential energy (1) is obtained if (i) $\text{Re}(Z)$ takes a VEV significantly larger than the self dual value of order unity:

$$\langle\text{Re}(Z)\rangle \sim \frac{1}{2\pi} \ln(m_{3/2}^2 M_P^2 / V_Q) \sim 32, \quad (6)$$

and (ii) (one-loop) threshold corrections to the gauge coupling constants not weaker than those of the standard model are Z -independent, which would assure that the quintessence axion does *not* couple to the instanton numbers generating a too large V_Q .²

¹Some aspects of the quintessence potential was discussed recently in [5].

²Of course this is true only when the tree level gauge coupling constants are also Z -independent.

As will be discussed in section IV, when combined with $\alpha_{GUT} = 1/25$ and $M_P = 2.4 \times 10^{18}$ GeV, the quintessence axion potential (5) which is presumed to take the value of (1) fixes all the couplings and scales of the underlying model. Particularly it gives the gauge unification scale

$$M_{GUT} \sim 1.3\gamma \times 10^{16} \left(\frac{32}{\langle \text{Re}(Z) \rangle} \right)^{1/2} \text{ GeV}, \quad (7)$$

where γ is a model-dependent constant of order unity which will be defined later. It is then interesting to note that $\langle \text{Re}(Z) \rangle \sim 32$ required for a quintessence axion determines M_{GUT} at a value close to the phenomenologically favored 3×10^{16} GeV.

One potential problem of the string or M -theoretic quintessence axion is that it does *not* satisfy the second slow-roll condition:

$$\left| \frac{\partial^2 V_Q}{\partial Q^2} \right| \lesssim \left| \frac{V_Q}{M_P^2} \right|. \quad (8)$$

As is well known, for the case of inflation in the early universe, this additional slow-roll condition is essential for the accelerated expansion goes over many Hubble times. Unless this condition is satisfied, the quintessence axion can lead to an accelerating universe at present *only* when it has a very particular initial value in the early universe, i.e. the quintessence axion suffers from the fine-tuning problem of initial condition. As we will see, the canonical quintessence axion field is given by

$$Q = M_P \sqrt{2\langle K_0'' \rangle} \text{Im}(Z), \quad (9)$$

where

$$K_0'' = \frac{\partial^2 K_0}{\partial Z \partial Z^*}$$

denotes the Kähler metric of the quintessence axion superfield Z . The slow-roll condition (4) applied for the axion potential (5) then leads to

$$|2\pi \text{Im}(Z)|_{\text{present}} \lesssim \frac{1}{2\pi} \sqrt{\langle 2K_0'' \rangle} \ll 1 \quad (10)$$

since it is expected that $\langle K_0'' \rangle \ll 1$ for the modulus VEV (6). This implies that at present the angular field $2\pi \text{Im}(Z)$ takes a value at near the top of its effective potential. Since V_Q violates the second slow-roll condition (8), viz

$$\left| \frac{\partial^2 V_Q}{\partial Q^2} \right| = \left| \frac{2\pi^2}{\langle K_0'' \rangle} \frac{V_Q}{M_P^2} \right| \gg \left| \frac{V_Q}{M_P^2} \right|, \quad (11)$$

and also $\text{Im}(Z) = 0$ corresponds to the maximum of V_Q , one needs a severe fine tuning of the initial values of Q and \dot{Q} in order to have the present value satisfying (10). It turns out that the degree of fine tuning is quite sensitive to $\langle K_0'' \rangle$:

$$|2\pi \text{Im}(Z)|_{\text{initial}} \lesssim \exp(-0.5m_0/H_0) \sim \exp(-3/\sqrt{\langle K_0'' \rangle}), \quad (12)$$

where $H_0 \approx V_Q/3M_P^2$ denotes the Hubble expansion rate at *present* and $m_0^2 = |\partial^2 V_Q/\partial Q^2| = |2\pi^2 V_Q/M_P^2 \langle K_0'' \rangle|$. If the Kähler metric of Z is dominated by the perturbative contribution, we have $\sqrt{\langle K_0'' \rangle} \approx 1/\langle \text{Re}(Z) \rangle \approx 3 \times 10^{-2}$. Such small value of the Kähler metric would lead to too severe fine tuning of the initial value of the quintessence axion, particularly when the quantum fluctuation of the axion field in the early universe is taken into account. However there can be a large *nonperturbative* contribution to the Kähler potential [8], which may be necessary to stabilize $\text{Re}(Z)$ at the value ~ 32 which is significantly bigger than the self dual value ~ 1 . If the stabilization of $\text{Re}(Z)$ is achieved by the nonperturbative effects encoded in the Kähler potential, the Kähler metric K_0'' would be dominated by the nonperturbative contribution, so that $\sqrt{\langle K_0'' \rangle}$ can be significantly bigger than the perturbative value 3×10^{-2} . In this case, the degree of the fine tuning of the initial conditions can be ameliorated enough to allow a cosmological scenario yielding the required initial condition (12). At the end of section IV, we discuss a late time inflation scenario based on the modular and CP invariance of the moduli effective potential, yielding the initial value of the quintessence axion satisfying (12) if the Kähler metric is enhanced by nonperturbative effects as

$$\sqrt{\langle K_0'' \rangle} \approx 9 \times 10^{-2}. \quad (13)$$

This scenario utilizes the specific property of string/M-theoretic quintessence axion $\text{Im}(Z)$ that its potential is of the order of $\bar{V} e^{-2\pi \text{Re}(Z)}$ where $\bar{V} = \mathcal{O}(H^2 M_P^2)$ over the period during which the expansion rate $H \gtrsim m_{3/2}$ and $\bar{V} = \mathcal{O}(m_{3/2}^2 M_P^2)$ for the subsequent period with $H \ll m_{3/2}$. This allows that $\text{Im}(Z)$ has a mass of the order of H during the late inflation with $H \sim m_{3/2}$ and $\text{Re}(Z) \sim 1$, while it becomes extremely light after the inflation since $\text{Re}(Z)$ is rolled down to a large value at the end of inflation. So during the late inflation, $\text{Im}(Z)$ rolls toward the minimum of the potential. If the axion potential is almost CP invariant, which would be the case when the scale of CP violation is low enough compared to M_P , the minimum of the axion potential during the late inflation would coincide with either the minimum or the maximum of the exponentially suppressed axion potential at the present. If it coincides to the maximum and also the Kähler metric is enhanced as (13), the latetime inflation can provide the desired initial condition (12) without any conflict with the known observational results.

II. EFFECTIVE POTENTIAL OF GENERIC QUINTESSENCE

Our starting point is the 4-dimensional $N = 1$ effective supergravity model [9] which is presumed to describe the dynamics of Q at high energy scales not far below M_P . The Kähler potential, superpotential and gauge kinetic functions of the model can be written as

$$\begin{aligned} K &= K_0(Z_i, Z_i^*) + Z_{\alpha\bar{\beta}}(Z_i, Z_i^*) C^\alpha C^{*\beta} + \left[\frac{1}{2} H_{\alpha\beta}(Z_i, Z_i^*) C^\alpha C^\beta \right. \\ &\quad \left. + \frac{1}{2} X_{\alpha\beta\bar{\gamma}}(Z_i, Z_i^*) C^\alpha C^\beta C^{*\gamma} + \text{h.c.} \right] + \dots, \\ W &= W_0(Z_i) + \frac{1}{3!} Y_{\alpha\beta\gamma}(Z_i) C^\alpha C^\beta C^\gamma + \frac{1}{4!} \Gamma_{\alpha\beta\gamma\delta}(Z_i) C^\alpha C^\beta C^\gamma C^\delta + \dots, \\ f_a &= f_{a0}(Z_i) + \frac{1}{2} F_{a\alpha\beta}(Z_i) C^\alpha C^\beta + \dots, \end{aligned} \quad (14)$$

where C^α denote the gauge-charged light matter superfields whose VEVs are far below M_P and Z_i stand for generic gauge-singlet light multiplets including the fields with large VEVs of $\mathcal{O}(M_P)$. Z_i may correspond to the string or M -theory moduli superfields and/or some gauge-invariant composite superfields made of strongly interacting hidden sector fields and/or others whose typical range of variation is of $\mathcal{O}(M_P)$. The quintessence Q is included in the model as a certain combination of Z_i and Z_i^* :

$$Q \in \{Z_i, Z_i^*\}. \quad (15)$$

The ellipses stand for the terms of higher order in C^α , and the leading non-renormalizable terms (in power countings of C^α) are explicitly written to illustrate their effects on V_Q . Note that unless specified we are using the standard supergravity unit with $M_P = 1$ throughout this paper.

The superpotential $W_0(Z_i)$ is assumed to be induced by some (nonperturbative) supersymmetry (SUSY) breaking dynamics, e.g. hidden sector gaugino condensation, which is already integrated out. The auxiliary F -components of certain Z_i develop non-zero VEVs:

$$F_i = \langle e^{K_0/2} D_i W_0 \rangle = \mathcal{O}(m_{3/2} M_P),$$

thereby breaking SUSY, where $D_i W_0 = \partial_i W_0 + W_0 \partial_i K_0$ is the Kähler covariant derivative with $\partial_i = \partial/\partial Z_i$. Here we concentrate on the supergravity-mediated SUSY breaking scenario [9] yielding the soft SUSY breaking parameters (the soft scalar masses, gaugino masses, e.t.c.) of order the gravitino mass $m_{3/2} = \langle e^{K_0/2} |W_0| \rangle$. For the case of gauge-mediation [10], the discussion will be modified but still leads to the same conclusion that K , W , and f_a should be invariant under the variation of quintessence, i.e. Q -independent, with an extremely fine accuracy. Note that unless constrained by some symmetries the coefficient functions in K , W , and f_a are generically Q -dependent, particularly when the terms of arbitrary order in Q are taken into account. Those coefficient functions determine the (nonrenormalizable) couplings of Q to the light gauge and charged matter multiplets. For instance, $Q^n F_{\mu\nu}^a F^{a\mu\nu}/M_P^n$ arises from a Q -dependent $\text{Re}(f_a)$, $Q^n F_{\mu\nu}^a \tilde{F}^{a\mu\nu}/M_P^n$ from a Q -dependent $\text{Im}(f_a)$, $Q^n C^\alpha \psi^\beta \psi^\gamma / M_P^n$ from a Q -dependent $Y_{\alpha\beta\gamma}$, and so on. At any rate, once the Q -invariance condition is fulfilled, non-derivative couplings of Q are highly suppressed, and as a result Q does not mediate any macroscopic force.

Obviously the tree level potential of Q can be determined by K_0 and W_0 via the conventional supergravity potential:

$$V_0 = e^{K_0} [K^{i\bar{j}} D_i W_0 D_{\bar{j}} W_0^* - 3|W_0|^2] \sim m_{3/2}^2 M_P^2, \quad (16)$$

where $K^{i\bar{j}}$ is the inverse of the Kähler metric $K_{i\bar{j}} = \partial_i \partial_{\bar{j}} K_0$ and the typical range of V_0 under the variation of $Z_i = \mathcal{O}(M_P)$ is estimated to be of order $m_{3/2}^2 M_P^2$. When quantum corrections are taken into account, generic (nonrenormalizable) couplings of Q to charged gauge and matter multiplets contribute to V_Q . The leading quantum correction comes from the quadratically divergent one-loop potential [11] including the piece depending on $Z_{\alpha\bar{\beta}}$ and $\text{Re}(f_a)$:

$$\delta V_1 = \frac{\Lambda^2}{16\pi^2} e^{K_0} D_i W_0 D_{\bar{j}} W_0^* R^{i\bar{j}} \sim \frac{1}{16\pi^2} m_{3/2}^2 \Lambda^2 \quad (17)$$

where the cutoff scale Λ is of order the messenger scale of SUSY breaking, which is essentially of $\mathcal{O}(M_P)$ for supergravity-mediated SUSY-breaking models,

$$R^{i\bar{j}} = K^{i\bar{k}} K^{l\bar{j}} \partial_l \partial_{\bar{k}} [\ln(\det Z_{\alpha\bar{\beta}}) - \ln(\det \text{Re}(f_{ab}))], \quad (18)$$

where $f_{ab} = f_a \delta_{ab}$, and the typical range of δV_1 is estimated to be of order $\frac{1}{16\pi^2} m_{3/2}^2 \Lambda^2$ for the variation of $Z_i = \mathcal{O}(M_P)$. This piece of one-loop potential arises from the well-known $V_1 = \frac{1}{16\pi^2} \Lambda^2 \text{Str}(M^2)$ where the mass matrix M^2 includes the soft SUSY breaking scalar and gaugino masses depending on $Z_{\alpha\bar{\beta}}$ and $\text{Re}(f_a)$ [11].

$H_{\alpha\beta}$ and $X_{\alpha\beta\bar{\gamma}}$ in K and the Yukawa couplings $Y_{\alpha\beta\gamma}$ in W affect the potential energy at two-loop order. It has been noted that the two-loop supergraph of Fig. 1 leads to the quadratically divergent Yukawa-dependent potential energy [12]:

$$\delta V_2 \sim \frac{\Lambda^2}{(16\pi^2)^2} e^{K_0} |DW_0|^2 |Y|^2 \sim \frac{|Y|^2}{(16\pi^2)^2} m_{3/2}^2 \Lambda^2, \quad (19)$$

where

$$\begin{aligned} |DW_0|^2 &= K^{i\bar{j}} D_i W_0 D_{\bar{j}} W_0^*, \\ |Y|^2 &= Z^{\alpha\bar{\alpha}} Z^{\beta\bar{\beta}} Z^{\gamma\bar{\gamma}} Y_{\alpha\beta\gamma} Y_{\bar{\alpha}\bar{\beta}\bar{\gamma}}^*. \end{aligned}$$

for $Z^{\alpha\bar{\beta}}$ being the inverse of $Z_{\alpha\bar{\beta}}$. It has been noted also that the Kähler metric ($Z_{\alpha\bar{\beta}}$) and soft SUSY-breaking scalar mass ($m_{\alpha\bar{\beta}}^2$) of C^α receive quadratically divergent corrections depending upon $H_{\alpha\beta}$ and $X_{\alpha\beta\bar{\gamma}}$ [13]:

$$\begin{aligned} \delta Z_{\alpha\bar{\beta}} &= -\frac{\Lambda^2}{16\pi^2} R_{\alpha\bar{\beta}}, \\ \delta m_{\alpha\bar{\beta}}^2 &= \frac{\Lambda^2}{16\pi^2} e^{K_0} |W_0|^2 R_{\alpha\bar{\beta}}, \end{aligned} \quad (20)$$

where

$$R_{\alpha\bar{\beta}} = Z^{\beta\bar{\alpha}} Z^{\gamma\bar{\gamma}} X_{\alpha\beta\bar{\gamma}} X_{\bar{\alpha}\bar{\beta}\bar{\gamma}}^* - Z^{\gamma\bar{\gamma}} K^{i\bar{j}} \partial_{\bar{j}} H_{\alpha\gamma} \partial_i H_{\bar{\beta}\bar{\gamma}}^*.$$

When combined together as the supergraph of Fig. 2, these one-loop corrections lead to the following quartically-divergent two-loop correction to the potential energy:

$$\begin{aligned} \delta V'_2 &\sim \left(\frac{\Lambda^2}{16\pi^2} \right)^2 e^{K_0} |W_0|^2 (|X|^2 + |\bar{\partial}H|^2) \\ &\sim \frac{|X|^2 + |\bar{\partial}H|^2}{(16\pi^2)^2} \frac{m_{3/2}^2 \Lambda^4}{M_P^2}, \end{aligned} \quad (21)$$

where

$$\begin{aligned} |X|^2 &= Z^{\alpha\bar{\alpha}} Z^{\beta\bar{\beta}} Z^{\gamma\bar{\gamma}} X_{\alpha\beta\bar{\gamma}} X_{\bar{\alpha}\bar{\beta}\bar{\gamma}}^* \\ |\bar{\partial}H|^2 &= Z^{\alpha\bar{\alpha}} Z^{\beta\bar{\beta}} K^{i\bar{j}} \partial_{\bar{j}} H_{\alpha\beta} \partial_i H_{\bar{\alpha}\bar{\beta}}^*. \end{aligned}$$

Finally $\Gamma_{\alpha\beta\gamma\delta}$ and $F_{\alpha\beta}$ contributes to the potential energy through the 3-loop supergraph of Fig. 3:

$$\begin{aligned}
\delta V_3 &\sim \frac{1}{(16\pi^2)^3} e^{K_0} (\Lambda^4 |DW_0|^2 |\Gamma|^2 + \Lambda^6 |\bar{D}W_0^* \partial F|^2) \\
&\sim \frac{1}{(16\pi^2)^3} \left(\frac{m_{3/2}^2 \Lambda^4}{M_P^2} |\Gamma|^2 + \frac{m_{3/2}^2 \Lambda^6}{M_P^4} |\partial F|^2 \right),
\end{aligned} \tag{22}$$

where $(\bar{D}W_0^* \partial F)_{\alpha\bar{\beta}} = K^{i\bar{j}} \partial_i F_{a\alpha\bar{\beta}} D_{\bar{j}} W_0^*$ and

$$\begin{aligned}
|\Gamma|^2 &= Z^{\alpha\bar{\alpha}} Z^{\beta\bar{\beta}} Z^{\gamma\bar{\gamma}} Z^{\delta\bar{\delta}} \Gamma_{\alpha\beta\gamma\delta} \Gamma_{\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\delta}}^*, \\
|\partial F|^2 &= K^{i\bar{j}} Z^{\alpha\bar{\beta}} Z^{\beta\bar{\alpha}} \partial_i F_{a\alpha\beta} \partial_{\bar{j}} F_{a\bar{\alpha}\bar{\beta}}^*.
\end{aligned} \tag{23}$$

So far, we have discussed the effective potential induced by the coefficient functions in K , W and f_a except for $\text{Im}(f_{0a})$ whose VEV corresponds to the vacuum angle of the a -th gauge group. In the effective supergravity model under consideration, possible nonperturbative hidden sector gauge interactions were already integrated out, and their effects are encoded in the effective superpotential W_0 which is a function of the holomorphic hidden sector gauge kinetic functions. This means that the effective potential of the both real and imaginary parts of the gauge kinetic functions of strongly interacting hidden sector are already included in (16).

Let us now consider the effective potential of the vacuum angles of the standard model gauge group. Due to the asymptotic freedom, the potential energy of the QCD vacuum angle $8\pi^2 \text{Im}(f_{QCD})$ is induced mainly by the low energy QCD dynamics at energy scales around 1 GeV [14], and thus is unambiguously determined as

$$V_{QCD} \sim f_\pi^2 m_\pi^2 \cos[8\pi^2 \text{Im}(f_{QCD})], \tag{24}$$

where m_π and f_π denote the pion mass and decay constant, respectively. However the potential energy of the electroweak vacuum angle $8\pi^2 \text{Im}(f_{EW})$ is mainly due to the $SU(2)_L$ instantons at high energy scales around M_{GUT} . Since the multi-fermion vertex of the standard model fermions induced by the electroweak instanton violates both the baryon (B) and lepton (L) numbers, $\Delta B = \Delta L = 3$, the resulting potential energy is suppressed by the insertions of small B and L -violating couplings in addition to the suppressions by the semiclassical factor $e^{-2\pi/\alpha_{GUT}}$ and the small SUSY breaking factor of order $m_{3/2}^2$. We thus have

$$V_{EW} \sim \epsilon e^{-2\pi/\alpha_{GUT}} m_{3/2}^2 M_P^2 \cos[\text{Im}(8\pi^2 f_{EW})], \tag{25}$$

where the small factor ϵ includes B and L violating couplings obeying the selection rule $\Delta B = \Delta L = 3$ together with small Yukawa-type couplings and loop factors which are necessary to close all fermion zero modes [15]. The size of ϵ is highly model-dependent, particularly on the B , L , and fermion chirality-changing couplings available at M_{GUT} .

Summing all the contributions discussed so far, the typical size of the quintessence potential V_Q is estimated to be

$$\begin{aligned}
V_Q &\sim m_{3/2}^2 M_P^2 \left[\frac{\delta \ln(K_0)}{\delta Q} + \frac{\delta \ln(W_0)}{\delta Q} + \frac{\Lambda^2}{16\pi^2 M_P^2} \left(\frac{\delta Z_{\alpha\bar{\beta}}}{\delta Q} + \frac{\delta \text{Re}(f_a)}{\delta Q} \right) \right. \\
&\quad \left. + \frac{\Lambda^2}{(16\pi^2)^2 M_P^2} \frac{\delta |Y|^2}{\delta Q} + \frac{\Lambda^4}{(16\pi^2)^2 M_P^4} \left(\frac{\delta |X|^2}{\delta Q} + \frac{\delta |\bar{\partial} H|^2}{\delta Q} \right) + \frac{\Lambda^4}{(16\pi^2)^3 M_P^4} \frac{\delta |\Gamma|^2}{\delta Q} \right]
\end{aligned}$$

$$+ \frac{\Lambda^6}{(16\pi^2)^3 M_P^6} \frac{\delta |\partial F|^2}{\delta Q} + \frac{f_\pi^2 m_\pi^2}{m_{3/2}^2 M_P^2} \frac{\delta \text{Im}(f_{QCD})}{\delta Q} + \epsilon e^{-2\pi/\alpha_{GUT}} \frac{\delta \text{Im}(f_{EW})}{\delta Q} \Big] Q_{\text{typ}}, \quad (26)$$

where δG denotes the variation of the coefficient function G for the quintessence variation δQ and Q_{typ} is the typical range of the quintessence variation. Note that (26) represents the variation of V_Q under the variation of Q specified by Q_{typ} . In order that $V_Q \sim (3 \times 10^{-3} \text{ eV})^4$ over the range of Q_{typ} , its variation should not significantly exceed $(3 \times 10^{-3} \text{ eV})^4$ over the same range. This requires that each term in (26) does not significantly exceed $(3 \times 10^{-3} \text{ eV})^4$ over the range of Q_{typ} . Note that although there may be a significant cancellation between different terms for a particular value of Q , it is extremely difficult that the cancellation persists over the range of Q_{typ} which is essentially of order M_P [5]. Requiring each term in (26) not exceed $(3 \times 10^{-3} \text{ eV})^4$, we find

$$\begin{aligned} \frac{\delta \ln(K_0)}{\delta Q} &\lesssim \frac{10^{-88}}{\kappa_Q \kappa_{3/2}}, & \frac{\delta \ln(W_0)}{\delta Q} &\lesssim \frac{10^{-88}}{\kappa_Q \kappa_{3/2}}, \\ \frac{\delta Z_{\alpha\bar{\beta}}}{\delta Q} &\lesssim \frac{10^{-81}}{\kappa_Q \kappa_\Lambda \kappa_{3/2}}, & \frac{\delta |\bar{\partial} H|^2}{\delta Q} &\lesssim \frac{10^{-74}}{\kappa_Q \kappa_\Lambda^2 \kappa_{3/2}}, \\ \frac{\delta |X|^2}{\delta Q} &\lesssim \frac{10^{-74}}{\kappa_Q \kappa_\Lambda^2 \kappa_{3/2}}, & \frac{\delta |Y|^2}{\delta Q} &\lesssim \frac{10^{-79}}{\kappa_Q \kappa_\Lambda \kappa_{3/2}}, \\ \frac{\delta |\Gamma|^2}{\delta Q} &\lesssim \frac{10^{-72}}{\kappa_Q \kappa_\Lambda^2 \kappa_{3/2}}, & \frac{\delta |\partial F|^2}{\delta Q} &\lesssim \frac{10^{-67}}{\kappa_Q \kappa_\Lambda^3 \kappa_{3/2}}, \\ \frac{\delta \text{Re}(f_a)}{\delta Q} &\lesssim \frac{10^{-81}}{\kappa_Q \kappa_\Lambda \kappa_{3/2}}, & \frac{\delta \text{Im}(f_{QCD})}{\delta Q} &\lesssim \frac{10^{-42}}{\kappa_Q \kappa_{3/2}}, \end{aligned} \quad (27)$$

where $\kappa_Q = Q_{\text{typ}}/M_P$, $\kappa_\Lambda = (\Lambda/10^{16} \text{ GeV})^2$, $\kappa_{3/2} = (m_{3/2}/\text{TeV})^2$, and all other quantities are defined in the supergravity unit with $M_P = 1$, e.g. Q , K_0 , W_0 , and $X_{\alpha\beta\bar{\gamma}}$ correspond to Q/M_P , K_0/M_P^2 , W_0/M_P^3 , and $M_P X_{\alpha\beta\bar{\gamma}}$, respectively. Note that at least one of the variations is required to saturate its upper limit to produce the desired quintessence potential energy.

When expanded in powers of C^α and also written in the unit with $M_P = 1$, all gauge-invariant coefficient functions in K , W , and f_a are expected to be of order unity except for W_0 which is of order $m_{3/2}$. Furthermore none of κ_Q , κ_Λ , and $\kappa_{3/2}$ can be small enough to significantly compensate the extremely small numerators in (27). Then the limits of (27) on the variations of coefficient functions imply that K , W and f_a *must be invariant under the variation of Q , i.e. Q -independent, with an extremely fine accuracy*. Note that terms of higher order in C^α which are not explicitly discussed here are similarly constrained to be Q -independent. At any rate, if this Q -invariance is satisfied, Q behaves like a (pseudo) Goldstone boson having only *derivative* couplings and thus does *not* mediate any macroscopic force.

A possible breaking of the Q -invariance whose size was not estimated in (27) is the variation of the electroweak vacuum angle $8\pi^2 \text{Im}(f_{EW})$ which is constrained as

$$\frac{\delta \text{Im}(f_{EW})}{\delta Q} \lesssim \frac{10^{-20}}{\epsilon \kappa_Q \kappa_{3/2}}. \quad (28)$$

Although quite sensitive to the physics at M_{GUT} , generically ϵ is suppressed by many powers of small Yukawa couplings, loop factors, and also small B and/or L -violating couplings, and

so it can be easily smaller than 10^{-20} [15]. This would suggest that $\text{Im}(f_{EW})$ can have a sizable Q -dependence, and so a sizable Q -coupling to the electromagnetic $F_{\mu\nu}\tilde{F}^{\mu\nu}$ which may lead to an interesting observable consequence [16]. However in view of the gauge coupling unification, i.e. $\langle \text{Re}(f_{QCD}) \rangle = \langle \text{Re}(f_{EW}) \rangle$, it is highly unlikely that $\text{Im}(f_{EW})$ has a sizable Q -dependence, while $\text{Im}(f_{QCD})$ is Q -independent as in (27).³

Let us close this section with a brief discussion of how plausible are some of the frequently used forms of V_Q [4] in view of our discussions above.

(a) Exponential potential: $V_Q \sim M^4 e^{-Q/f_Q}$. This form of V_Q may arise from some nonperturbative dynamics when Q corresponds to a dilaton [19]. At certain level, one may adjust the dynamics of the model to make this exponential potential to be of order $(3 \times 10^{-3} \text{ eV})^4$. However usually the gauge coupling constants in the model are determined by the dilaton VEV, which means $\delta \text{Re}(f_a)/\delta Q$ is of order unity. Then the supergravity loop effects depending on $\delta \text{Re}(f_a)/\delta Q$ (see Eq.(26)) yield a dilaton potential energy of order $\frac{1}{16\pi^2} m_{3/2}^2 \Lambda^2$ which is too large to be a quintessence potential energy.

(b) Inverse power law potential: $V_Q \sim M^{4+k}/Q^k$. This form of V_Q may arise from a nonperturbative dynamics when Q corresponds to a composite degree of freedom [19]. An attractive feature of this potential is that M does not have to be too small compared to the particle physics mass scales, e.g. $M \gtrsim 10 \text{ GeV}$ for $k \geq 3$ and $Q \sim M_P$. However the inverse power law behavior is so easily upset either by the supersymmetry breaking dynamics yielding the soft mass term $\sim m_{3/2}^2 Q^2$ or by the Planck scale dynamics which would generate a non-renormalizable term $\delta V_Q \sim m_{3/2}^2 M_P^2 Q^n / M_P^n$. If any of such terms is induced, which is extremely difficult to avoid, the variation of V_Q under $\delta Q \sim M_P$ would be too large to provide a quintessence potential.

(3) Pseudo-Goldstone boson potential: $V_Q \sim M^4 \cos(Q/f_Q)$. This form of V_Q arises when Q corresponds to a (pseudo) Goldstone boson, which is perhaps the most plausible possibility in view of the Q -invariance conditions of (27) [17]. Obviously then the Q -invariance corresponds to the nonlinearly realized global symmetry associated with the Goldstone boson Q . Still the remained question is how the explicit breaking of the non-linear global symmetry can be so tiny as in (27), which is very nontivial to achieve in view of that global symmetries are generically broken by Planck scale physics [20]. In the next section, we argue that heterotic M -theory or Type I string axion can be a plausible candidate for the quintessence Goldstone boson with the tiny Q -invariance breaking induced by the membrane or D -brane instantons.

³A possible loophole [17] for this argument is the possibility of massless up quark which has been argued to be phenomenologically viable [18]. In this case, the effective potential of the QCD vacuum angle $\text{Im}(f_{QCD})$ is suppressed by $z = m_u/m_d$, i.e. $V_{QCD} \sim z f_\pi^2 m_\pi^2$, and then the upper bound on $\delta \text{Im}(f_{QCD})/\delta Q$ in Eq.(27) should be read as $10^{-42}/z \kappa_Q \kappa_\Lambda$. Thus for $|z| \lesssim 10^{-40}$, $\text{Im}(f_{QCD})$ can have a sizable Q -dependence and so Q can couple to both $(F\tilde{F})_{QCD}$ and $(F\tilde{F})_{EW}$.

III. HETEROTIC M OR TYPE I STRING AXION AS QUINTESSENCE

The most natural candidate for a light scalar field whose typical range of variation is of order M_P is the string or M -theory moduli multiplets describing the (approximately) degenerate string or M -theory vacua. It is thus quite tempting to look at the possibility that Q corresponds to a certain combination of the string or M -theory moduli superfields. In this section, we examine this possibility and point out that the heterotic M -theory or Type I string axion can be a plausible candidate for quintessence. The moduli superfields of our interests are $Z_I = (S, T_i)$ including the axion components $\text{Im}(Z_I)$ together with the modulus components $\text{Re}(Z_I)$ which correspond to the string dilaton or the length of the 11-th segment and the Kähler moduli describing the size and shape of the internal 6-manifold. Explicit computations show that generically

$$\frac{\delta K_0}{\delta \text{Re}(Z_I)} = \mathcal{O}(1), \quad (29)$$

and so the effective potential of the modulus components are of order $m_{3/2}^2 M_P^2$ and the modulus masses are of order $m_{3/2}$ in view of the analysis in the previous section. It thus appears that the modulus components $\text{Re}(Z_I)$ can *not* provide a quintessence. However as we will see still quintessence can arise as a linear combination of the axion components $\text{Im}(Z_I)$ if the corresponding modulus component has a large VEV ~ 32 .

Let us first consider the heterotic M -theory on a 11-dimensional manifold with boundary which is invariant under the Z_2 -parity [6]:

$$C \rightarrow -C, \quad x^{11} \rightarrow -x^{11}, \quad (30)$$

where $C = C_{ABC} dx^A dx^B dx^C$ is the 3-form field in the 11-dimensional supergravity. When compactified to 4-dimensions, axions arise as the massless modes of $C_{\mu\nu 11}$ and $C_{mn 11}$:

$$\epsilon^{\mu\nu\rho\sigma} \partial_{[\nu} C_{\rho\sigma]11} = \partial^\mu \eta_S, \quad C_{mn 11} = \sum_i \eta_i(x^\mu) \omega_{mn}^i, \quad (31)$$

where ω^i ($i = 1$ to $h_{1,1}$) form the basis of the integer $(1,1)$ cohomology of the internal 6-manifold and μ, ν are tangent to the noncompact 4-dimensional spacetime. In 4-dimensional effective supergravity, these axions appear as the pseudo-scalar components of chiral multiplets:

$$S = (4\pi)^{-2/3} \kappa^{-4/3} V + i\eta_S, \\ T_i = (4\pi)^{-1/3} \kappa^{-2/3} \int_{\mathcal{C}_i} \omega \wedge dx^{11} + i\eta_i, \quad (32)$$

where κ^2 denotes the 11-dimensional gravitational coupling, V is the volume of the internal 6-manifold with the Kähler two form ω , and the integral is over the 11-th segment and also over the 2-cycle \mathcal{C}_i dual to ω^i . Here the axion components are normalized by the discrete Peccei-Quinn (PQ) symmetries:

$$\text{Im}(S) \rightarrow \text{Im}(S) + 1, \quad \text{Im}(T_i) \rightarrow \text{Im}(T_i) + 1, \quad (33)$$

which are the parts of discrete modular symmetries involving also the dualities between large and small $\text{Re}(S)$ or $\text{Re}(T_i)$.

Holomorphy and the discrete PQ symmetries imply that in the large $\text{Re}(S)$ and $\text{Re}(T_i)$ limits the gauge kinetic functions can be written as⁴

$$4\pi f_a = k_a S + \sum_i l_{ai} T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}), \quad (34)$$

where k_a and l_{ai} are model-dependent *quantized* real constants and the exponentially suppressed terms are possibly due to the membrane or 5-brane instantons. For a wide class of compactified heterotic M -theory, we have [22–26]

$$\begin{aligned} 4\pi f_{E_8} &= S + \sum_i l_i T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}), \\ 4\pi f_{E'_8} &= S - \sum_i l_i T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}), \end{aligned} \quad (35)$$

where $l_i T_i$ corresponds to the one-loop threshold correction in perturbative heterotic string terminology with the quantized coefficients l_i determined by the instanton numbers on the hidden wall and also the orbifold twists.⁵

Let Q denote a linear combination of $\text{Im}(T_i)$ for which the combination $\sum_i l_i T_i$ is *invariant* under

$$U(1)_Q : \quad Q \rightarrow Q + \text{constant}. \quad (36)$$

Note that such Q exists always as long as $h_{1,1} > 1$. In this regard, models of particular interest are the recently discovered threshold-free models with $l_i = 0$ [26] for which any of $\text{Im}(T_i)$ can be identified as Q . At any rate, once such Q exists, we have

$$\frac{\delta f_a}{\delta Q} = \mathcal{O}(e^{-2\pi T}), \quad (37)$$

where the internal 6-manifold is assumed to be isotropic and thus $\text{Re}(T_i) \approx \text{Re}(T)$ for all T_i . Holomorphy and discrete PQ symmetries imply also

$$\frac{\delta Y_{\alpha\beta\gamma}}{\delta Q} = \mathcal{O}(e^{-2\pi T}) \quad (38)$$

for the Yukawa couplings (and also higher order holomorphic couplings) in the superpotential of (14). If W_0 is induced by hidden sector dynamics described by Q -*independent* gauge, Yukawa, and higher order holomorphic couplings, we have

⁴ This feature of the gauge kinetic function was first noted in [21].

⁵ For compactifications on smooth Calabi-Yau space, l_i can be determined either in the perturbative heterotic string limit [27,22] or in the 11-dimensional supergravity limit [28,25] and they take half-integer values [24], while they can be generic rational numbers for orbifolds [26].

$$\frac{\delta W_0}{\delta Q} = \frac{\delta W_0}{\delta f_a} \frac{\delta f_a}{\delta Q} + \frac{\delta W_0}{\delta Y_{\alpha\beta\gamma}} \frac{\delta Y_{\alpha\beta\gamma}}{\delta Q} + \dots = \mathcal{O}(e^{-2\pi T} W_0). \quad (39)$$

About the Kähler potential, the discrete PQ symmetries (33) imply that it can be written as $K = \sum_n K_n \exp[i2\pi n \text{Im}(T_i)]$ where K_n is a function of $\text{Re}(T_i)$ and other moduli (and also of charged matter superfields). Obviously then $U(1)_Q$ is broken by K_n with $n \neq 0$. To estimate the size of K_n , it is convenient to consider the 4-dimensional $N = 1$ vacuum as a deformation of $N = 2$ vacuum *keeping* the discrete PQ symmetries unbroken and also $\text{Re}(T_i)$ at large values [24]. Note that this deformation is not a shift within the $N = 2$ moduli space, but involves the shifts of massive fields breaking $N = 2$ supersymmetry down to $N = 1$ supersymmetry. At $N = 2$ vacuum, T_i correspond to the coordinates of a special Kähler manifold whose Kähler potential is determined by the *holomorphic* prepotential [29,30], and thus the discrete PQ symmetries imply that $K_n \propto e^{-2\pi n T_i}$. As long as the discrete PQ symmetries are kept unbroken and also $\text{Re}(T_i)$ kept large, this exponential suppression of the $U(1)_Q$ -breaking is expected to be maintained when $N = 2$ vacuum is deformed to $N = 1$ vacua of phenomenological interests, implying that

$$\frac{\delta K}{\delta Q} = \mathcal{O}(e^{-2\pi T}) \quad (40)$$

even at $N = 1$ vacua.⁶

When applied for the effective potential analysis of the previous section, Eqs. (37), (38), (39), and (40) assure that in the large $\text{Re}(T)$ limit the quintessence axion potential is exponentially suppressed as

$$V_Q \sim e^{-2\pi \langle \text{Re}(T) \rangle} m_{3/2}^2 M_P^2 \cos[2\pi \text{Im}(T)] \quad (41)$$

even when all possible quantum corrections are taken into account. Thus what is necessary and sufficient for this axion potential to be of order $(3 \times 10^{-3} \text{ eV})^4$ is

$$\langle \text{Re}(T) \rangle \sim \frac{1}{2\pi} \ln(m_{3/2}^2 M_P^2 / V_Q) \sim 32, \quad (42)$$

where $m_{3/2} \sim 1 \text{ TeV}$ are used for numerical estimate.⁷ As we will see in the next section, this value of $\text{Re}(T)$ can be compatible with $\alpha_{GUT} \sim 1/25$ *only* in the heterotic M -theory limit, *not* in the perturbative heterotic string limit. Although the above quintessence

⁶ This conclusion is supported by the observation that the nonlinear PQ transformation $U(1)_i : \delta T_i = i c_i$ ($c_i = \text{real constant}$) originates from the *local* transformation of the 11-dimensional three form field, $\delta C = \omega^i \wedge dx^{11} \partial_{11} \beta^i(x^{11})$ with $c_i = \int \delta C = \int \omega^i \wedge dx^{11} \partial_{11} \beta^i$. As was noted in [22], this suggests that $U(1)_i$ is broken *only* by nonperturbative effects defined on the boundaries or by those involving the bulk degree of freedoms wrapping the 2-cycle \mathcal{C}_i dual to ω^i and stretched between the boundaries. Since $U(1)_Q$ was chosen to avoid the breaking by gauge instantons on the boundaries, it appears to be broken only by the stretched membrane instantons wrapping \mathcal{C}_i whose effects are suppressed by $e^{-2\pi T_i}$.

⁷ About stabilizing $\text{Re}(T)$ at this large value, it has been shown in [31] that both $\text{Re}(S)$ and $\text{Re}(T)$ can be stabilized at large VEVs of order $\frac{1}{\alpha_{GUT}}$ by the combined effects of the multi-gaugino condensations and the membrane instantons wrapping the 3-cycle of the internal 6-manifold.

axion potential has been obtained by the macroscopic analysis based on supersymmetry and discrete PQ symmetries, one can easily identify its microscopic origin by noting that $2\pi\text{Re}(T_i) = (4\pi)^{-1/3}\kappa^{-2/3}\int_{\mathcal{C}_i}\omega\wedge dx^{11}$ corresponds to the Euclidean action of the membrane instanton wrapping \mathcal{C}_i and stretched along the 11-th segment [22]. When extrapolated to the perturbative heterotic string limit, such membrane instanton corresponds to the heterotic string worldsheet instanton wrapping the same 2-cycle [32]. Explicit computations then show that the Kähler potential and/or the gauge kinetic functions are indeed corrected by worldsheet instantons, yielding $\delta K_0 = \mathcal{O}(e^{-2\pi T})$ and $\delta f_a = \mathcal{O}(e^{-2\pi T})$ [33]. These corrections can be smoothly extrapolated back to the heterotic M -theory limit and identified as the corrections induced by stretched membrane instantons. When integrated out, the correction to the hidden sector gauge kinetic function leaves its trace in the effective superpotential as $\delta W_0 = \mathcal{O}(e^{-2\pi T}W_0)$. The axion potential (41) is then obtained from the supergravity potential (16) with $\delta K_0 = \mathcal{O}(e^{-2\pi T})$ and/or $\delta W_0 = \mathcal{O}(e^{-2\pi T}W_0)$.

Let us now turn to Type I string axions. The Type I axions (again normalized by the discrete PQ symmetries of (33)) correspond to the massless modes of the R-R two form fields $B_{\mu\nu}$ and B_{mn} , $\epsilon^{\mu\nu\rho\sigma}\partial_\nu B_{\rho\sigma} = \partial^\mu\eta_S$, $B_{mn} = \sum_i \eta_i(x^\mu)\omega_{mn}^i$, and form 4-dimensional chiral multiplets together with the string dilaton e^D and the internal space volume V :

$$\begin{aligned} S &= (2\pi)^{-6}\alpha'^{-3}e^{-D}V + i\eta_S, \\ T_i &= (2\pi)^{-2}\alpha'^{-1}e^{-D}\int_{\mathcal{C}_i}\omega + i\eta_i. \end{aligned} \quad (43)$$

Here we include $D9$ and $D5$ branes in the vacuum configuration, and consider the 4-dimensional gauge couplings α_9 and α_{5i} defined on $D9$ branes wrapping the internal 6-manifold and $D5$ branes wrapping the 2-cycles \mathcal{C}_i , respectively [34]. Again holomorphy and discrete PQ symmetries imply that the corresponding gauge kinetic functions can be written as (34). A simple leading order calculation gives $\alpha_9 = 1/\text{Re}(S)$ and $\alpha_{5i} = 1/\text{Re}(T_i)$, and so [34]

$$4\pi f_9 = S, \quad 4\pi f_{5i} = T_i. \quad (44)$$

However this leading order result can receive perturbative and/or non-perturbative corrections. Generic perturbative corrections can be expanded in powers of the string coupling e^D , the string inverse tension α' , and also the inverse tension $e^D\alpha'^k$ of D_{2k-1} branes [7]. Generically they scale as

$$e^{nD}\alpha'^m \propto [\text{Re}(T)]^{\frac{n+m}{2}}[\text{Re}(S)]^{-\frac{3n+m}{2}}. \quad (45)$$

Combined with the leading order result (44) and also the general form of gauge kinetic function (34) dictated by holomorphy and discrete PQ symmetries, this scaling behavior implies that f_9 can receive a T_i -dependent correction at order α'^2 , while there is no perturbative correction to f_{5i} , and so

$$\begin{aligned} 4\pi f_9 &= S + l_i T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}), \\ 4\pi f_{5i} &= T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}). \end{aligned} \quad (46)$$

Similarly to the case of heterotic M -theory axion, the quintessence Q can arise as a linear combination of $\text{Im}(S)$ and $\text{Im}(T_i)$, however its explicit form depends on how the

gauge couplings *not* weaker than those of the standard model are embedded in the model. Note that for such *not-so-weak* gauge couplings both $\text{Re}(f_a)$ and $\text{Im}(f_a)$ are required to be Q -independent as in (27), while for the weaker gauge interactions with $\alpha_a(M_{GUT}) \lesssim 1/32$, $\text{Im}(f_a)$ are allowed to have a sizable Q -dependence. Here are some possibilities. If all of the not-so-weak gauge couplings are embedded in α_{5i} , $\text{Im}(S)$ can be a quintessence when $\langle \text{Re}(S) \rangle \sim 32$ which is necessary for $V_Q \sim (3 \times 10^{-3} \text{ eV})^4$, and also the associated gauge coupling $\alpha_9(M_{GUT}) \lesssim 1/32$ which may be necessary to avoid a too large V_Q induced by the gauge instantons of α_9 . If not-so-weak couplings are embedded in α_9 , any of $\text{Im}(T_i)$ can be a quintessence when $\langle \text{Re}(T_i) \rangle \sim 32$. If the vacuum does not include any $D5$ brane wrapping the particular 2-cycle, e.g. the i -th cycle, the corresponding axion $\text{Im}(T_i)$ can be a quintessence independently of the embedding when $\langle \text{Re}(T_i) \rangle \sim 32$.

Once the quintessence component satisfying the constraints on gauge kinetic functions is identified, the Q -dependence of the Kähler potential and superpotential is suppressed by $e^{-2\pi Z}$ ($Z = S$ or T) as in the case of heterotic M -theory, and so the quintessence axion potential is again given by (41) where now T can be either S or T_i . Microscopic origin of this Type I axion potential can be easily identified also by noting that $2\pi\text{Re}(S)$ corresponds to the Euclidean action of $D5$ brane instanton wrapping the internal 6-manifold and $2\pi\text{Re}(T_i)$ is of $D1$ string instanton wrapping the 2-cycle \mathcal{C}_i .

IV. COUPLINGS AND SCALES WITH QUINTESSENCE AXION

In the previous section, it was noted that the heterotic M or Type I string axion can provide a quintessence dark energy when its modulus partner has a VEV ~ 32 . In this section, we discuss the couplings and scales associated with this modulus VEV.

A. Couplings and Scales in Heterotic M -theory:

For simplicity, let us consider a model without T -dependent threshold corrections, i.e. the quantized coefficients $l_i = 0$ in Eq. (35). We then have [28,22]

$$\begin{aligned} \frac{1}{\alpha_{GUT}} &= (4\pi)^{-2/3} \kappa^{-4/3} V = \text{Re}(S), \\ \omega &= \frac{(4\pi)^{1/3} \kappa^{2/3}}{\pi\rho} \sum_i \text{Re}(T_i) \omega^i, \\ V &= \frac{1}{3!} \int \omega \wedge \omega \wedge \omega \approx \frac{4\pi\kappa^2}{(\pi\rho)^3} \frac{\sum_{ijk} C_{ijk}}{6} [\text{Re}(T)]^3 \end{aligned} \quad (47)$$

where $\pi\rho$ is the length of the 11-th segment, ω is the Kähler form of the internal 6-manifold, and $C_{ijk} = \int \omega^i \wedge \omega^j \wedge \omega^k$ are the intersection numbers of the integer $(1,1)$ cohomology basis ω^i . Here we have used the isotropy condition $\text{Re}(T_i) \approx \text{Re}(T)$. Combining these with

$$M_P^2 = \pi\rho V / \kappa^2,$$

one easily finds

$$\begin{aligned} M_{KK} &\equiv \frac{\gamma}{V^{1/6}} = \frac{\gamma M_P}{2\sqrt{\pi} (\sum C_{ijk}/6)^{1/6} [\text{Re}(S)]^{1/2} [\text{Re}(T)]^{1/2}} \\ \kappa^{-2/3} (\pi\rho)^3 &= (4\pi)^{1/3} \left(\frac{\sum C_{ijk}}{6} \right) \frac{[\text{Re}(T)]^3}{\text{Re}(S)} \end{aligned} \quad (48)$$

where M_{KK} denotes the Kaluza-Klein threshold mass scale for the internal 6-manifold with volume V . Here γ is a constant of order unity whose precise value depends on the details of compactification and also on the regularization of the threshold effects due to massive Kaluza-Klein modes. (For instance, γ is expected to be around 2π for toroidal compactifications, however it can be smaller for more complicate compactifications, e.g. about $\sqrt{2\pi}$ for the compactification on S^6 .)

In heterotic M -theory, M_{KK} can be identified as the unification scale of gauge couplings on the boundary [28]:

$$M_{GUT} = M_{KK}.$$

Also the VEVs of $\text{Re}(S)$ and $\text{Re}(T)$ can be determined by the two phenomenological inputs: $\alpha_{GUT} = 1/25$ and $V_Q = (3 \times 10^{-3} \text{ eV})^4$, yielding

$$\begin{aligned} \text{Re}(S) &= \frac{1}{\alpha_{GUT}} \sim 25, \\ \text{Re}(T) &= \frac{1}{2\pi} \ln(m_{3/2}^2 M_P^2 / V_Q) \sim 32. \end{aligned}$$

Applying these moduli VEVs to (48), we find

$$M_{GUT} = \frac{1.3\gamma}{(\sum C_{ijk}/6)^{1/6}} \times 10^{16} \text{ GeV}, \quad (49)$$

which is very close to the phenomenologically favored value $3 \times 10^{16} \text{ GeV}$. We stress that the large $\langle \text{Re}(T) \rangle \sim 32$ required for the quintessence axion is essential for M_{GUT} to have a value close to the phenomenologically favored value. Large $\text{Re}(T)$ leads also to the length of the 11-th segment significantly bigger than the 11-dimensional Planck length:

$$\pi\rho \sim 10\kappa^{2/9}, \quad (50)$$

implying that we are indeed in the 11-dimensional heterotic M -theory limit corresponding to the strong coupling limit of the heterotic $E_8 \times E'_8$ string theory.

B. Couplings and Scales in Type I string with D9 and D5-branes:

Again for simplicity consider a model without T -dependent threshold correction to f_9 in (46). We then have [7]

$$\begin{aligned} M_P^2 &= 2(2\pi)^{-7} \alpha'^{-4} e^{-2D} V, \\ \frac{1}{\alpha_9} &= (2\pi)^{-6} \alpha'^{-3} e^{-D} V = \text{Re}(S), \\ \frac{1}{\alpha_{5i}} &= (2\pi)^{-2} \alpha'^{-1} e^{-D} \int_{C_i} \omega = \text{Re}(T_i), \end{aligned} \quad (51)$$

and so

$$\begin{aligned} e^{2D} &= \frac{\text{Re}(S)}{(\sum_{ijk} C_{ijk}/6) [\text{Re}(T)]^3}, \\ M_{KK} &\equiv \frac{\gamma}{V^{1/6}} = \frac{\gamma M_P}{2\sqrt{\pi} (\sum C_{ijk}/6)^{1/6} [\text{Re}(S)]^{1/2} [\text{Re}(T)]^{1/2}}, \\ M_{string} &\equiv \frac{1}{\sqrt{\alpha'}} = \frac{\sqrt{\pi} M_P}{(\sum C_{ijk}/6)^{1/2} [\text{Re}(S)]^{1/4} [\text{Re}(T)]^{3/4}}. \end{aligned} \quad (52)$$

In Type I case with a quintessence axion $\text{Im}(T)$ or $\text{Im}(S)$, we also have

$$\begin{aligned} M_{GUT} &= M_{KK} \text{ or } M_{string}, \\ \frac{1}{\alpha_{GUT}} &= \text{Re}(S) \text{ or } \text{Re}(T) \sim 25, \\ \frac{1}{2\pi} \ln(m_{3/2}^2 M_P^2 / V_Q) &= \text{Re}(T) \text{ or } \text{Re}(S) \sim 32, \end{aligned}$$

depending upon how the standard model gauge couplings are embedded [34]. For these moduli VEVs,

$$e^{2D} \ll 1, \quad \alpha' M_{KK}^2 = \mathcal{O}(1), \quad (53)$$

indicating that we are in a domain of weakly coupled string but of strongly coupled world-sheet sigma model. It is also straightforward to see that they also give M_{GUT} which is very close to the phenomenologically favored value, similarly to the case of heterotic M -theory.

C. Quintessence Axion Scale and the Fine Tuning Problem of Initial Condition:

Let $K_0(Z + Z^*)$ denote the (dimensionless) Kähler potential of the quintessence axion multiplet $Z = T$ or S . When all massive moduli (including $\text{Re}(Z)$) are integrated out, the effective lagrangian of the superlight quintessence axion is given by

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= M_P^2 \langle K_0'' \rangle [\partial_\mu \text{Im}(Z)]^2 + m_{3/2}^2 M_P^2 e^{-2\pi \langle \text{Re}(Z) \rangle} \cos[2\pi \text{Im}(Z)] \\ &= \frac{1}{2} (\partial_\mu Q)^2 + m_{3/2}^2 M_P^2 e^{-2\pi \langle \text{Re}(T) \rangle} \cos(Q/f_Q) \end{aligned} \quad (54)$$

where the canonical quintessence axion Q and its decay constant f_Q are given by

$$Q = M_P \sqrt{2 \langle K_0'' \rangle} \text{Im}(Z), \quad f_Q = \frac{1}{2\pi} M_P \sqrt{2 \langle K_0'' \rangle} \quad (55)$$

with the Kähler metric $K_0'' = \partial^2 K_0 / \partial Z \partial Z^*$.

In order to have negative pressure, Q must roll slowly and so satisfy

$$\left| \frac{\partial V_Q}{\partial Q} \right| \lesssim \left| \frac{V_Q}{M_P} \right|. \quad (56)$$

When applied for (54), this slow-roll condition leads to

$$\begin{aligned} |2\pi \text{Im}(Z)|_{\text{present}} &\lesssim \sqrt{2 \langle K_0'' \rangle} / 2\pi, \\ |2\pi \text{Im}(\dot{Z})|_{\text{present}} &\lesssim H_0 / \sqrt{2 \langle K_0'' \rangle}, \end{aligned} \quad (57)$$

implying that at present the angular field $2\pi \text{Im}(Z)$ should be at near the top of its effective potential unless the Kähler metric K_0'' is significantly bigger than the unity. (Here H_0 denotes the present Hubble expansion rate.)

At leading order approximation in string or M theory, we have [7]

$$K_0 \approx -c \ln(Z + Z^*), \quad (58)$$

where c is a constant of order unity. Obviously this gives a small Kähler metric $K_0'' \sim 1/[\text{Re}(Z)]^2 \ll 1$ for $\text{Re}(Z) \sim 32$. This then leads to

$$\left| \frac{\partial^2 V_Q}{\partial Q^2} \right| = \left| \frac{2\pi^2}{\langle K_0'' \rangle} \frac{V_Q}{M_P^2} \right| \gg \left| \frac{V_Q}{M_P^2} \right|, \quad (59)$$

and thus the quintessence axion in string or M theory largely violates the second slow-roll condition:

$$\left| \frac{\partial^2 V_Q}{\partial Q^2} \right| \lesssim \left| \frac{V_Q}{M_P^2} \right|. \quad (60)$$

As a consequence, the quintessence axion can lead to an accelerating universe at present, i.e. can satisfy Eq. (57), *only* when it has a very particular initial value in the early universe, i.e. the quintessence axion suffers from the fine-tuning problem of the initial conditions of Q and \dot{Q} in the early universe. As long as $\sqrt{\langle K_0'' \rangle}/\pi \ll 1$, the corresponding quintessence axion requires the fine tuning of initial condition, however the degree of required fine tuning is quite sensitive to the value of $\sqrt{\langle K_0'' \rangle}$. Although $K_0'' \ll 1$ at leading order perturbation theory, it can be significantly enhanced by $\text{Im}(Z)$ -*independent* nonperturbative effects which would be responsible for stabilizing $\text{Re}(Z)$ at large value ~ 32 [8]. In this case, the fine tuning problem of initial conditions would be ameliorated, however it is hard to imagine that nonperturbative effects are strong enough to give $K_0'' \sim \pi^2$, so that allow to avoid the fine tuning of initial condition.

The above discussed fine tuning problem of initial conditions appears to be a serious difficulty of the quintessence axion. Here we present a late time inflation scenario which would resolve this difficulty for a reasonably enlarged value of the Kähler metric $\langle K_0'' \rangle$. The gauge symmetries of string or M -theory include discrete modular group [7] under which Z and other generic moduli Φ transform as

$$\text{Re}(Z) \rightarrow \frac{1}{\text{Re}(Z)}, \quad \text{Im}(Z) \rightarrow \text{Im}(Z) + 1, \quad \Phi \rightarrow \Phi', \quad (61)$$

and also CP [35] under which

$$Z \rightarrow Z^*, \quad \Phi \rightarrow \Phi^*. \quad (62)$$

Here we take the simplest form of the Z -duality ($Z = S$ or T), i.e. $\text{Re}(Z) \rightarrow 1/\text{Re}(Z)$ with the self-dual value $\text{Re}(Z) = 1$, however our discussion is valid for other forms of the Z -duality transformation as long as the self-dual value is of order unity.

The modular and CP invariances assure that the invariant points,

$$\text{Re}(Z) = 1, \quad \text{Im}(Z) = 0 \text{ or } \frac{1}{2}, \quad \Phi = \Phi^* \text{ or } \Phi'^*, \quad (63)$$

correspond to the stationary points of the effective action [36]. It is then quite possible that $Z = 1$ was a local minimum of the effective potential *during the inflation period* if the inflaton is moduli invariant field. At this inflationary minimum, all the moduli masses have the same order of magnitude:

$$m_{\text{Re}(Z)} \sim m_{\text{Im}(Z)} \sim m_\Phi \sim H_{\text{inf}}, \quad (64)$$

where $H_{\text{inf}} = \sqrt{V_{\text{inf}}/M_P^2}$ is the expansion rate during inflation. Note that the axion mass of $m_{\text{Im}(Z)}$ is comparable to other moduli masses for $\langle \text{Re}(Z) \rangle = \mathcal{O}(1)$. The modulus $\text{Re}(Z)$ can have appropriate couplings to inflaton field, so that if inflaton takes its present value, $\text{Re}(Z) = 1$ becomes an unstable point of the moduli potential. Then after the inflation $\text{Re}(Z)$ moves toward the *present minimum* at $\text{Re}(Z) \sim 32$ for which

$$m_{\text{Re}(Z)} \sim m_{3/2}, \quad m_{\text{Im}(Z)} \sim e^{-\pi \langle \text{Re}(Z) \rangle} m_{3/2}. \quad (65)$$

About the location of the minimum in the axion direction, we have just two possibilities if CP is *not* spontaneously broken in the moduli sector. One of the two CP invariant points, $\text{Im}(Z) = 0$ and $1/2$, is the minimum, while the other corresponds to the maximum. Our key assumption on the axion potential is that $\text{Im}(Z) = 0$ is the minimum for the inflationary modulus value $\text{Re}(Z) \sim 1$, however it becomes the maximum for the present value $\text{Re}(Z) \sim 32$. Note that the coefficient of the cosine potential of $\text{Im}(Z)$ is a function of $\text{Re}(Z)$, and so its sign can be changed when $\text{Re}(Z)$ varies from the inflationary value to the present value.

Given the properties of the moduli potential discussed above, the cosmological scenario yielding the present values of $\text{Im}(Z)$ and $\text{Im}(\dot{Z})$ satisfying Eq. (57) goes as follows. There was a relatively late inflationary period in the early universe during which $\text{Re}(Z)$ and other moduli were settled down near at the modular and CP-invariant local minimum of the potential, i.e. at $\langle \text{Re}(Z) \rangle_{\text{inf}} = 1$, $\langle \text{Im}(Z) \rangle_{\text{inf}} = 0$, and $\langle \Phi \rangle_{\text{inf}} = \langle \Phi^* \rangle_{\text{inf}}$. Since the modulus $\text{Re}(Z)$ is significantly away from the present value ~ 32 , it is expected that the Hubble expansion rate H_{inf} of this late inflation is *at least* of order $m_{3/2}$. To avoid a too large quantum fluctuation of the quintessence axion, we assume the minimal value

$$H_{\text{inf}} = \mathcal{O}(m_{3/2}). \quad (66)$$

During the inflation period, all moduli masses including that of $\text{Im}(Z)$ are of order H_{inf} , and thus all moduli are settled down at the values sufficiently near at the minimum if the number of inflation efoldings (N_e) are large enough [37]. In particular, just after the inflation, we have

$$\begin{aligned} |2\pi \text{Im}(Z)|_{\text{inflation}} &= \mathcal{O}(e^{-3N_e/2}) + \mathcal{O}(H_{\text{inf}}/M_P), \\ |2\pi \text{Im}(\dot{Z})|_{\text{inflation}} &= \mathcal{O}(e^{-3N_e/2} H_{\text{inf}}) + \mathcal{O}(H_{\text{inf}}^2/M_P), \end{aligned} \quad (67)$$

where the exponential suppression in the right hand side is due to the classical evolution toward the minimum of $V_{\text{inflation}}$, while the second terms denote the effects of quantum fluctuations. After the inflation, $\text{Re}(Z)$ moves toward the present value $\text{Re}(Z) \sim 32$. After $\text{Re}(Z)$ is settled down at its present value, $\text{Im}(Z) = 0$ becomes the maximum of the axion potential with

$$m_0 \equiv \left| \frac{\partial^2 V_Q}{\partial Q^2} \right|_{Q=0}^{1/2} \approx \frac{\sqrt{3}\pi}{\sqrt{\langle K_0'' \rangle}} H_0. \quad (68)$$

Since $m_0 \gg H_0$, the inflationary value $\text{Im}(Z) \sim 0$ is highly unstable against the cosmological evolution during the period with $H_0 \lesssim H \lesssim m_0$. The resulting instability factor is given

by $e^{\alpha m_0/H_0}$ where α is a constant of order unity whose precise value depends upon the initial conditions. More detailed study [38] suggests $\alpha \approx 0.5$ for the most of interesting initial conditions. In order for the quintessence axion to provide an accelerating universe at present, we then need

$$\begin{aligned} |2\pi\text{Im}(Z)|_{\text{inflation}} &\lesssim e^{-0.5m_0/H_0} \sim e^{-3/\sqrt{\langle K_0'' \rangle}}, \\ |2\pi\text{Im}(\dot{Z})|_{\text{inflation}} &\lesssim H_{\text{inf}} e^{-0.5m_0/H_0} \sim H_{\text{inf}} e^{-3/\sqrt{\langle K_0'' \rangle}}. \end{aligned} \quad (69)$$

In order for the initial condition (67) set by the late time inflation to satisfy the above condition, we need first a large efolding:

$$N_e \gtrsim \frac{2}{\sqrt{\langle K_0'' \rangle}}, \quad (70)$$

and also a small quantum fluctuation of the quintessence axion field during (and also after) the late inflation:

$$\delta Q \sim \frac{H_{\text{inf}}}{2\pi} \sim \frac{m_{3/2}}{2\pi} \lesssim M_P e^{-3/\sqrt{\langle K_0'' \rangle}}. \quad (71)$$

In fact, N_e can not be arbitrarily large. For late time inflation with $H_{\text{inf}} \sim m_{3/2}$, it is very difficult to generate the observed density fluctuation $\delta\rho/\rho \sim 10^{-5}$. It is thus reasonable to assume that density fluctuations were created before the late inflation. Then the late inflation is required not to destroy the pre-existing density fluctuations. This consideration gives an upper bound of N_e [37], yielding

$$N_e \lesssim 25 \sim 30 \quad (72)$$

for a weak scale $m_{3/2}$ and the reheat temperature range $T_r = 10^5 \sim 10^{-2}$ GeV.

The above conditions (70), (71) and (72) can not be simultaneously satisfied for the perturbative value of $\langle K_0'' \rangle$. At leading order in perturbation theory, we have $K_0 \sim 3 \ln(Z + Z^*)$ and thus $\sqrt{\langle K_0'' \rangle} \sim 1/\langle \text{Re}(Z) \rangle \sim 3 \times 10^{-2}$ for $\langle \text{Re}(Z) \rangle \sim 32$, which is obviously in conflict with the above conditions. However as was noted in the [8], there can be a rather large *nonperturbative* contribution to the Kähler potential which may be essential for stabilizing $\text{Re}(Z)$ at the desired value ~ 32 . In particular, if the nonperturbative terms involve a large power of $(Z + Z^*)$, e.g. $K_{np} = h(Z + Z^*)^k e^{-b\sqrt{Z+Z^*}} = \mathcal{O}(1)$ with $k \gtrsim 6$, the Kähler metric can be significantly enlarged to a value

$$\sqrt{\langle K_0'' \rangle} \approx 9 \times 10^{-2} \quad (73)$$

which would accomodate all the conditions for the quintessence axion to lead to an accelerating universe at present.

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FIGURES

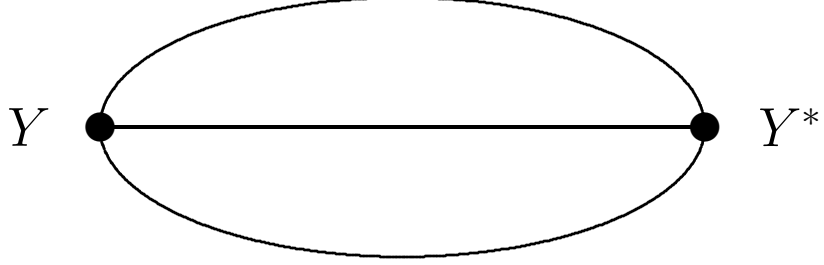


FIG. 1. Two loop supergraph inducing the quadratically-divergent potential energy of $Y_{\alpha\beta\gamma}$.

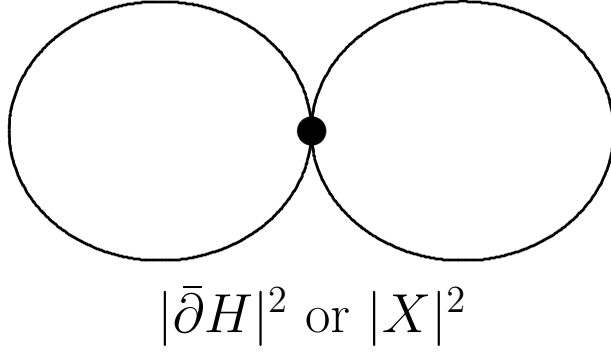


FIG. 2. Quartically-divergent two loop supergraph for the potential energy of $H_{\alpha\beta}$ and $X_{\alpha\beta\bar{\gamma}}$.

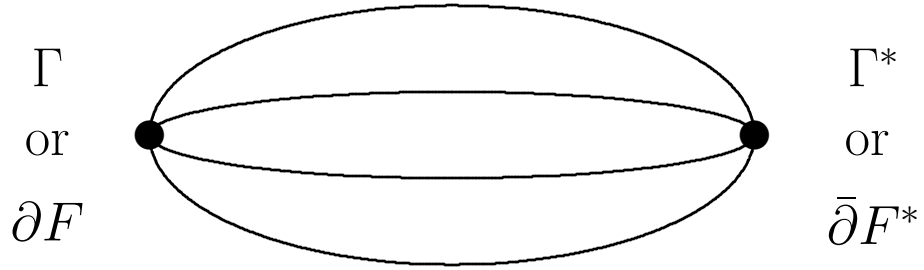


FIG. 3. Three loop supergraph for the potential energy of $\Gamma_{\alpha\beta\gamma\delta}$ and $F_{a\alpha\beta}$. For $F_{a\alpha\beta}$, two lines denote the gauge multiplets while the others are the charged matter multiplets.